Optimal Sensing Strategy Using Spatially Averaged Advection-Diffusion Parameter Estimation

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E N G I N E E R I N G

Motivation

Develop an algorithm to estimate the parameters of an advection-diffusion process using mobile sensing agents for applications such as:

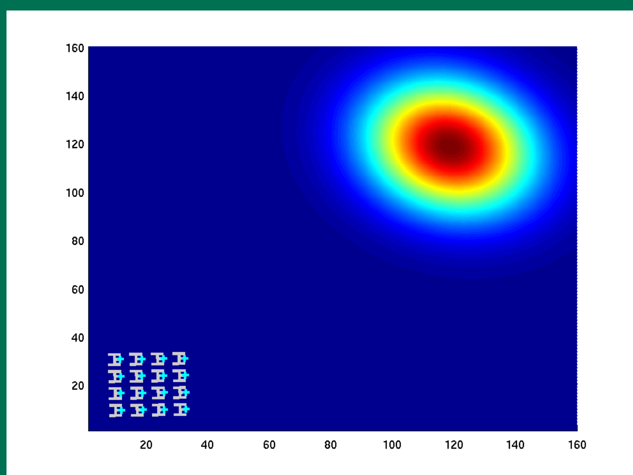
- Environmental monitoring
- Harmful algal bloom tracing
- Chemical plume tracing

Algorithm

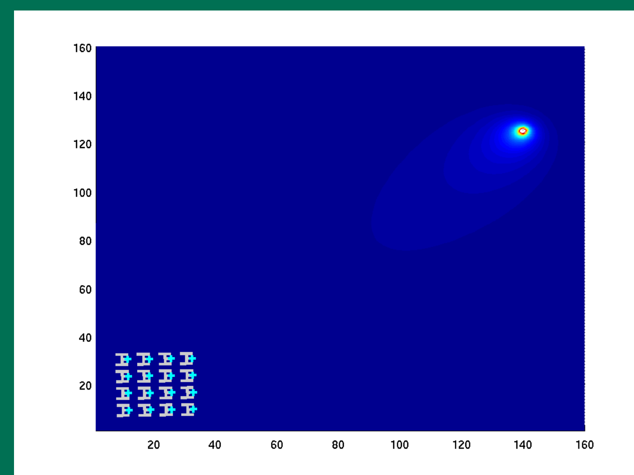
- Each mobile sensing agent takes noisy measurements of the concentration field at its current location.
- Each mobile sensing agent shares its measurements with the leader agent.
- Nonlinear linear least squares method is applied to the collected measurements to estimate the parameters.
- The sensing agents are then driven in the direction that increases the quality of the estimated parameters.

Advection-Diffusion Process

- The concentration field considered corresponds to a closed-form solution of a simple advection-diffusion process.
  - Two advection-diffusion processes are considered



Impulse Response



Continuous Source

Advection-Diffusion Process

- Impulse Response Function

$$C(x,y,t) = \frac{C_0}{\sqrt{(4\pi)^3\,(t-t_0)^3(D_{xx}D_{yy}-D_{xy}^2)}}\exp\Bigg\{-\frac{1}{4(D_{xx}D_{yy}-D_{xy}^2)\,(t-t_0)}$$

$$\times\ \left[(\vec{x}-\vec{x}_0-\vec{V}(t-t_0))\right]^{\mathsf{T}}\begin{bmatrix} D_{yy} & -D_{xy} \\ -D_{xy} & D_{xx}\end{bmatrix}\left[(\vec{x}-\vec{x}_0-\vec{V}(t-t_0))\right]\Bigg\}$$

Goal: Recover $\theta = [D_{xx} \quad D_{yy} \quad Dxy \quad C_0 \quad x_0 \quad y_0 \quad t_0]^{\mathsf{T}}$

- Continuous Source

$$C(x,y) = \frac{q}{2\pi K d_s}\exp\left(-\frac{U}{2K}(d_s-\triangle x)\right),$$

where

$$d_s = \sqrt{(x_0-x)^2+(y_0-y)^2},$$
$$\triangle x = (x_0-x)\cos\theta + (y_0-y)\sin\theta.$$

Goal: Recover $\theta = [K \quad q \quad x_0 \quad y_0]^{\mathsf{T}}$

Gradient Control Strategy

- The advection-diffusion process parameters are estimated by utilizing nonlinear least squares optimization on the collected measurements.

- To quantify the information that the concentration measurements carry about the unknown parameters the Fisher Information Matrix is used

$$\mathcal{I} := \frac{\Sigma(\theta^\star)}{\sigma^2}, \text{ with } \Sigma := \sum_{i=1}^{n} C'(\mathcal{Q}_i, \theta^\star) C'(\mathcal{Q}_i, \theta^\star)^T$$

where $C'(\mathcal{Q}_i, \theta) := \left[\frac{\partial}{\partial \theta_j} C(\mathcal{Q}_i, \theta) \right]_j$

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Gradient Control Strategy

- Since the true parameter $\theta^\star$ is not known, the current estimates $\hat{\theta}$ are used to compute $\mathcal{I}$.
- To improve the quality of the parameter estimates $\hat{\theta}$, the determinant of $\mathcal{I}$ is maximized.

$$\mathcal{J} = \det \mathcal{I} = \frac{1}{\sigma^2} \det \Sigma$$

- The objective function $\mathcal{J}$ is maximized by steering the sensing agents so that they climb the gradient of $\mathcal{J}$ with respect to $\vec{x}$.

$$\begin{aligned} \frac{\partial \mathcal{J}}{\partial \vec{x}} &= \sum_{ij} \left(\frac{\partial \mathcal{J}(\Sigma)}{\partial \Sigma} \right) \left(\frac{\partial \Sigma}{\partial \vec{x}} \right)_k \\ &= \sum_{ij} \left(\det(\Sigma)(\Sigma)^{-T} \right) \left(\frac{\partial \Sigma(\vec{x})}{\partial \vec{x}} \right)_k \end{aligned}$$

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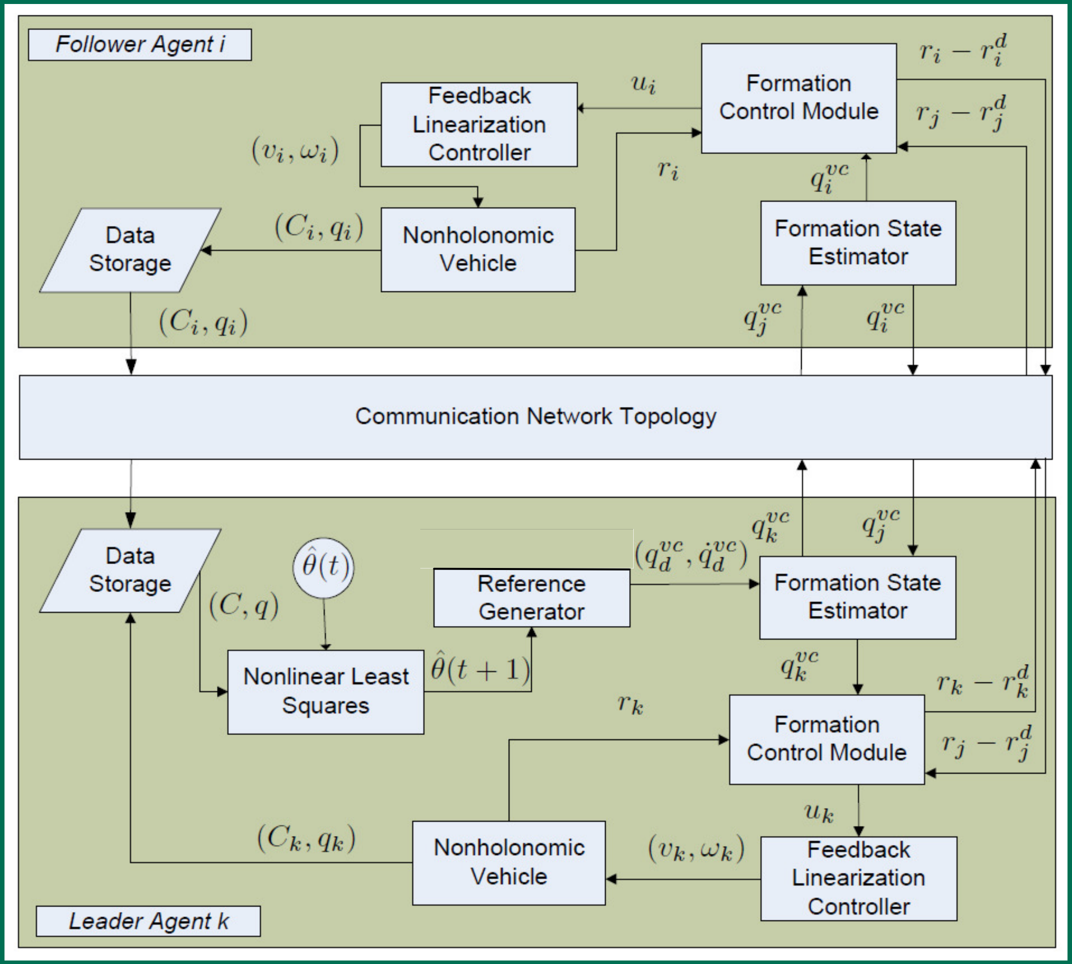
Gradient Control Strategy

- The gradient control is then defined as

$$\dot{q}_d^{vc} = \begin{cases} v & \text{if } \|v\| < v_{sat}, \\ \frac{v}{\|v\|} v_{sat} & \text{if } \|v\| \geq v_{sat}, \end{cases}$$

where $v = \varepsilon \frac{\partial \mathcal{J}}{\partial \vec{x}}$, v_{sat} is the saturation velocity and $\varepsilon > 0$ is a gain.

Flow Chart of Optimal Sampling Strategy



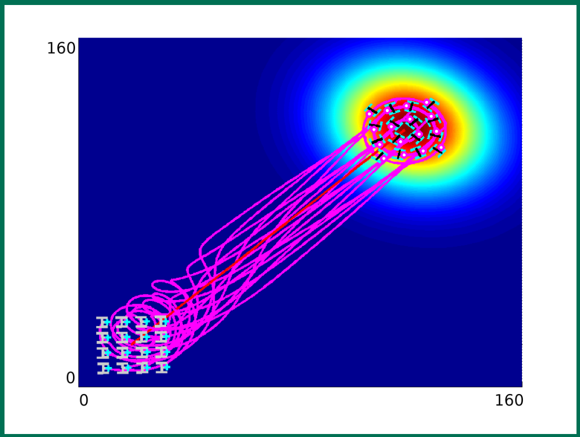
Simulation Results

Table of parameters used in the impulse response simulation.

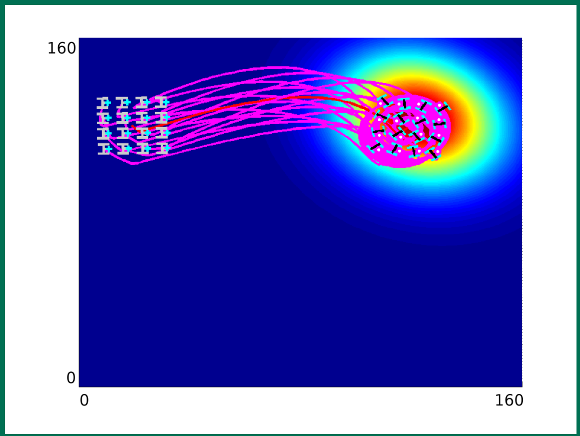
Parameters	True Values	apriori Values	Estimated Values with starting position (x_S, y_S) $(20, 20)$	$(120, 20)$	$(20, 120)$
D_{xx} $\left(\frac{m^2}{s}\right)$	0.07	0.084	0.0699	0.0703	0.0702
D_{yy} $\left(\frac{m^2}{s}\right)$	0.07	0.084	0.0697	0.0703	0.0702
D_{xy} $\left(\frac{m^2}{s}\right)$	-0.01	-0.012	-0.0099	-0.0101	-0.0101
C_0 $\left(\frac{kg}{m^3}\right)$	5.0×10^6	6.0×10^6	5.0043×10^6	5.0149×10^6	5.0124×10^6
(x_0, y_0) (m)	$(80, 80)$	$(1, 1)$	$(79.93, 79.89)$	$(79.93, 80.08)$	$(80.03, 79.95)$
t_0 (s)	30.0	36.0	24.2717	30.2394	29.4507

Simulation Results

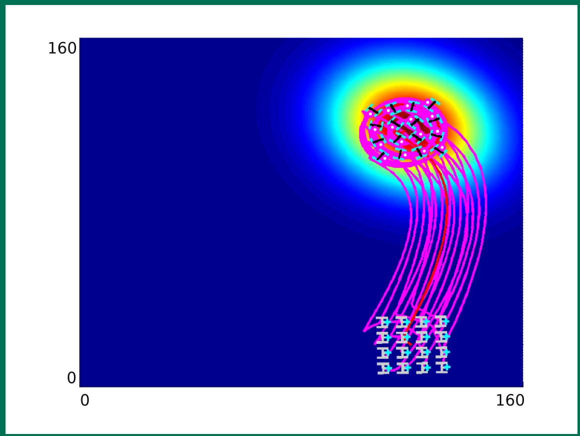
Figures of the sensing agents starting at three different positions.



$(x_s, y_s) = (20, 20)$



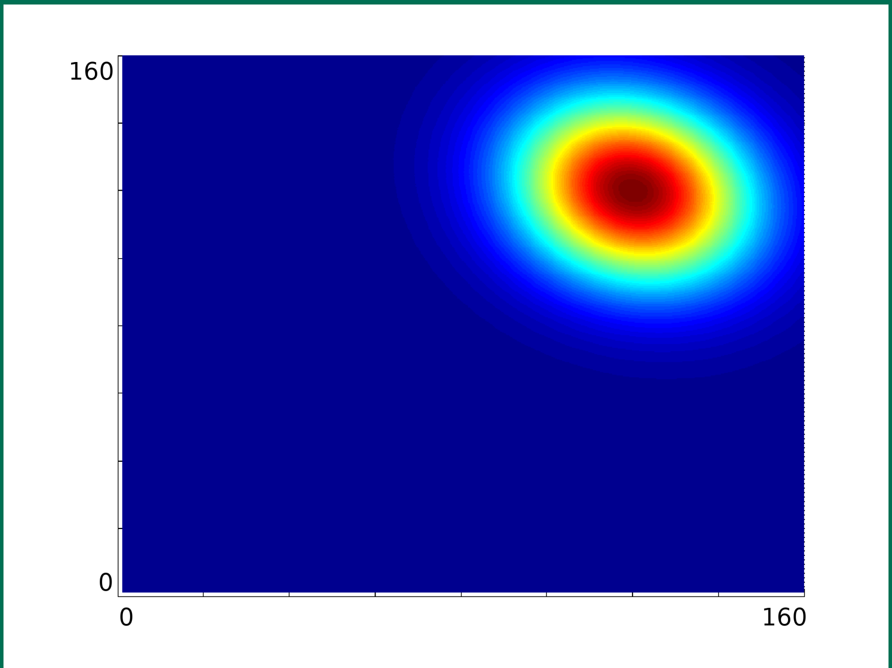
$(x_s, y_s) = (20, 120)$



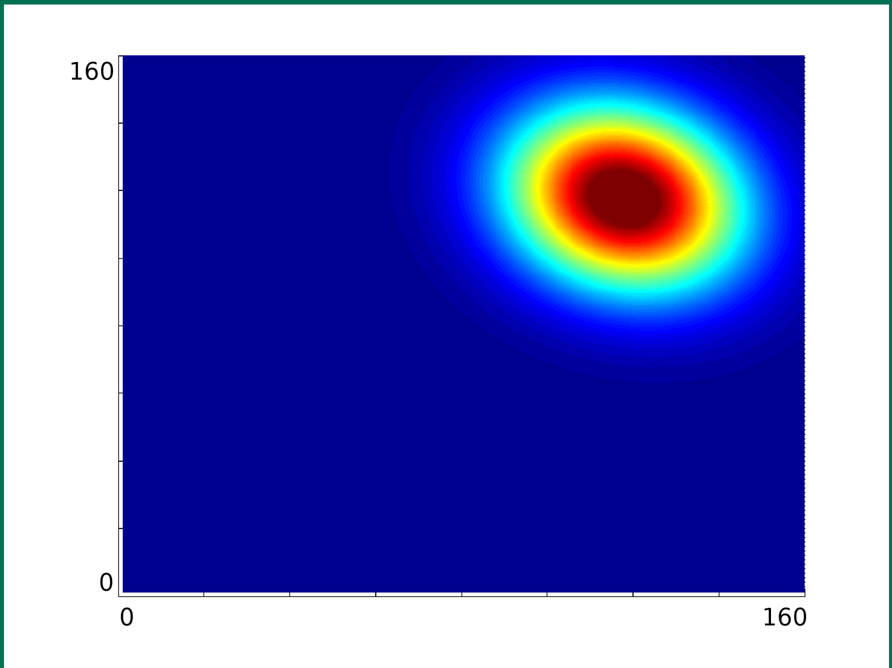
$(x_s, y_s) = (120, 20)$

Simulation Results

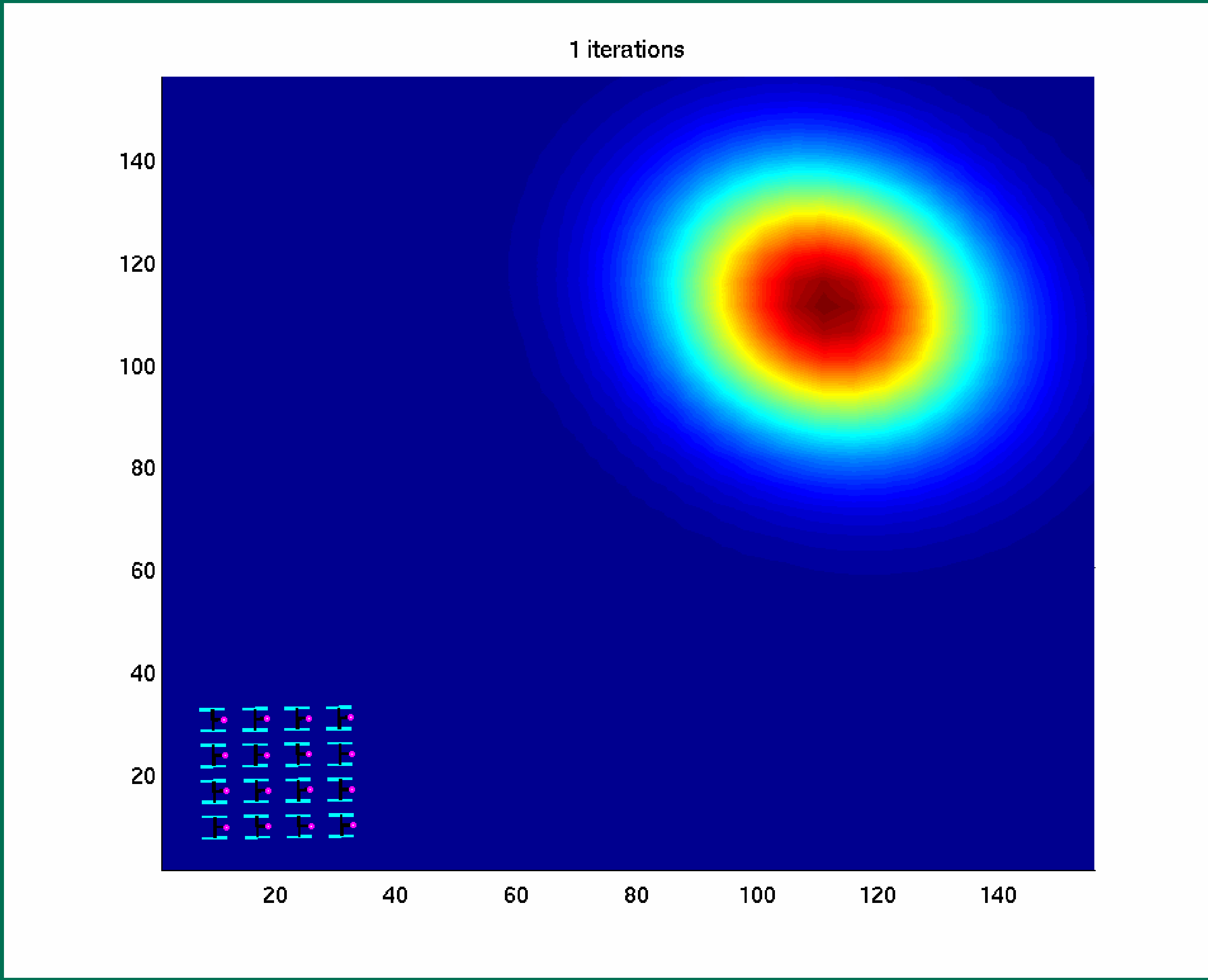
Comparison of the true and estimated concentration fields.



True Concentration Field



Estimated Concentration Field


1 iterations
140
120
100
80
60
40
20
20
40
60
80
100
120
140

Simulation Results

Table of parameters used in the continuous source simulation.

Parameters	True Values	apriori Values	Estimated Values with starting position (x_s, y_s) $(20, 20)$	$(120, 20)$	$(20, 120)$
$K\ \left(\frac{m^2}{s}\right)$	50.0	60.0	228.54	1000000.0	68.75
$q\ \left(\frac{m^2}{s}\right)$	5000.0	6000.0	17877.52	69315759.6	6018.19
$(x_0, y_0)\ (m)$	(140, 125)	(1, 1)	(138.86, 124.46)	(121.41, 122.51)	(139.51, 124.84)

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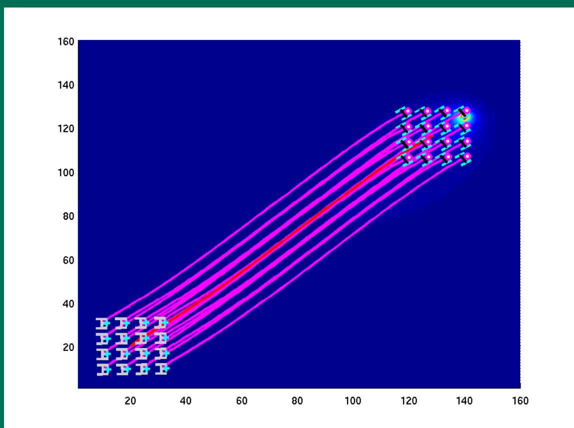
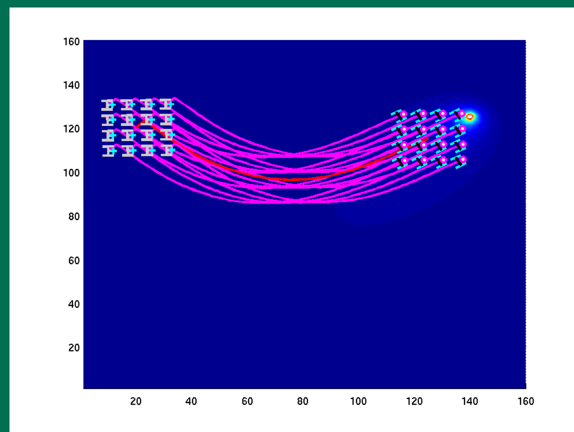
Simulation Results

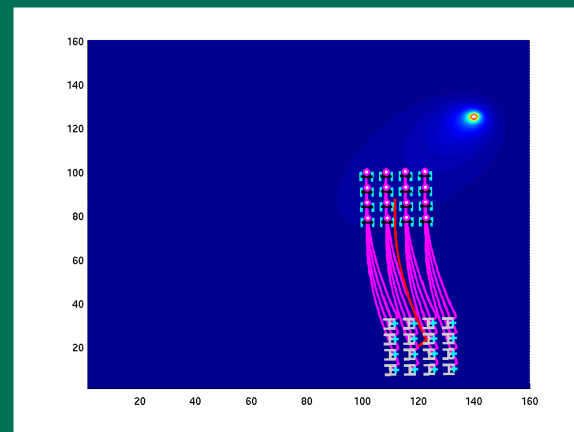
Figures of the sensing agents starting at three different positions.



$(x_s, y_s) = (20, 20)$



$(x_s, y_s) = (20, 120)$



$(x_s, y_s) = (120, 20)$

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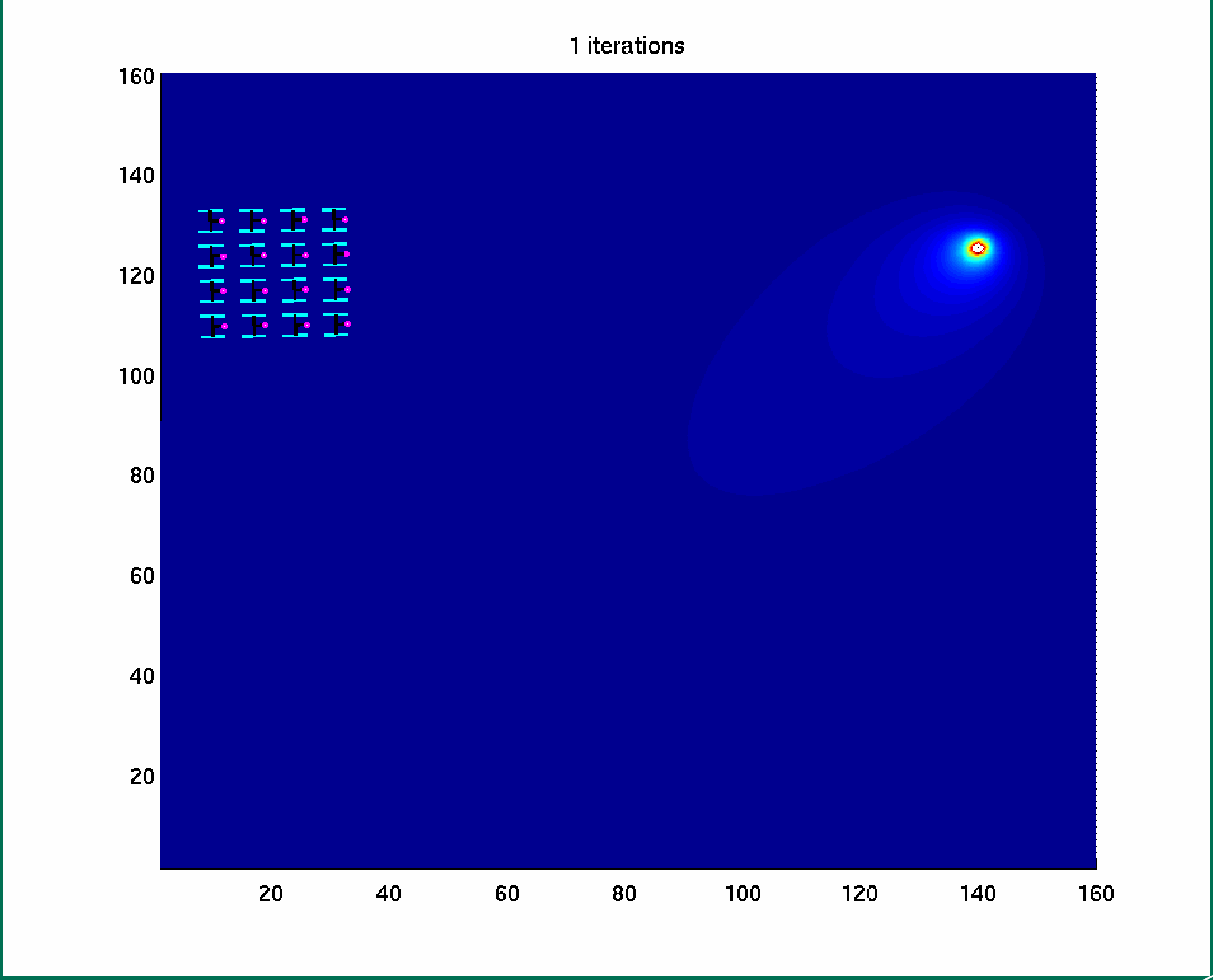

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Questions???

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